q-space MR propagator in partially-restricted, axially-symmetric and isotropic environments

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INTRODUCTION

Diffusion of molecules is a powerful indicator of tissue microstructure. The dependence of the MR signal on the acquisition parameters (e.g., the magnitude and orientation of the diffusion gradients) can be exploited to infer valuable information from tissue. One way to elucidate microstructural features of the specimen involves fitting physical models to such q-space data. Alternatively, the diffusion-weighted MR signal profile can be transformed into a probability distribution function quantifying the average probability for molecular displacements. We refer to this function as the apparent propagator. The reconstructed apparent propagator was shown to be a valuable marker of tissue microstructure [1]. For example, in three-dimensional q-space acquisitions, the maxima of the orientation-dependent propagator have been associated with the orientations of white-matter fibers in the brain [2].

In this work, our focus is somewhat different than characterizing the microstructural features of tissue, although our findings are expected to be used in such studies. Instead, in more general terms, we discuss the q-space MR propagators and propose three new ways of obtaining them. We illustrate the utility of these new definitions of the apparent propagator on diffusion taking place in simple geometries such as partially-restricted environments, curving fibers and spherical compartments.

THEORY & RESULTS

In non-imaging MR acquisitions, since the protons precess in the clockwise direction on the plane perpendicular to the main magnetic field, an inverse Fourier transformation of the MR signal attenuations should be employed, leading to a displacement probability function consistent with the notion of an ensemble average propagator. However, when one is interested in quantifying a displacement distribution in every voxel of an MR image, the propagator derived in the traditional way may lead to a counter-intuitive profile when it is non-symmetric. By exploiting the reciprocity of the real diffusion propagator, $K(\mathbf{x}_1,\mathbf{x}_2,\Delta)=K(\mathbf{x}_2,\mathbf{x}_1,\Delta)$, an alternative is introduced, which implies a forward Fourier transform of the MR signal attenuations, $E(\mathbf{q},\Delta)$, yielding a propagator reflected around the origin. We denote the propagators obtained in the traditional and new ways by $P_{\mathrm{im},1}(\mathbf{X},\mathbf{u})$ and $P_{\mathrm{im},2}(\mathbf{X},\mathbf{u})$, respectively, which are given by

$$P_{\mathrm{im},1}(\mathbf{X},\mathbf{u}) = \int E(\mathbf{X},\mathbf{q})\, e^{i2\pi\mathbf{q}\cdot\mathbf{u}} \mathrm{d}\mathbf{q} = \rho_{\mathrm{N}}(\mathbf{X}-\mathbf{u},\mathbf{X})\, K(\mathbf{X}-\mathbf{u},\mathbf{X},\Delta), \text{ and } P_{\mathrm{im},2}(\mathbf{X},\mathbf{u}) = \int E(\mathbf{X},\mathbf{q})\, e^{-i2\pi\mathbf{q}\cdot\mathbf{u}} \mathrm{d}\mathbf{q} = \rho_{\mathrm{N}}(\mathbf{X}+\mathbf{u},\mathbf{X})\, K(\mathbf{X},\mathbf{X}+\mathbf{u},\Delta).$$

Here, $\mathbf{X}$, $\mathbf{u}$ and Δ denote the location of the voxel, displacement vector, and the diffusion time, respectively. ρ_{N} is a normalized dimensionless spin density function. Clearly, the expressions on the right-hand-side of the above equations suggest that $P_{\mathrm{im},1}(\mathbf{X},\mathbf{u})$ is the probability that a particle will undergo a displacement $\mathbf{u}$ *before* it arrives at the voxel location $\mathbf{X}$. In contrast, the second notion of the propagator quantifies the probability that a particle situated at the voxel location will have moved a distance $\mathbf{u}$ *away* from the voxel location.

As an example for a partially restricted environment in which the two definitions of the apparent propagator differ, we consider a very simple geometry where diffusion is impeded by an infinite impermeable plate located at X=0, restricting the diffusing particles to the region X>0 as shown in Figure1. Here, $\omega=(4D_0\Delta)^{1/2}$, where D_0 is the diffusivity. Following the treatment in [3], we can analytically evaluate the MR signal attenuation from which the two notions of the apparent propagators can be computed. It is clear that the propagator $P_{\mathrm{im},1}(X,u)$ leads to a counter-intuitive picture in which displacements towards the wall appear to be finite whereas displacements to the right are terminated although the wall is to the left of the voxel. The second notion of the propagator, however, has the more intuitive shape as the displacements through the wall are cut off.

Another interesting example that we consider is diffusion taking place inside curving fibers. To simulate such a system, we take a circular loop whose thickness is assumed to be infinitesimal for simplicity. We shall suppose that our images are of dimension 2x2, where each pixel contains one of the quadrants of the loop yielding a fiber curving towards a different direction in each pixel (see Figure 2). To evaluate the signal and the associated apparent propagators, we employed the true propagator, which is obtained by using the method in [4]. The long diffusion time propagators are illustrated in Figure 2. Clearly, in this case, the first notion of the apparent propagator is more intuitive in that the bright rims are consistent with the curvature of the fibers.

Finally, we consider environments possessing axial-symmetry or isotropy, in which case having data along one-direction in q-space is sufficient to reconstruct two- and three-dimensional propagators, respectively. The relevant propagators are given by the following sine and Hankel transforms:

$$P_{\mathrm{3D}}(R) = \frac{2}{R}\int_0^\infty E(q)\sin(2\pi qR)q\,\mathrm{d}q, \text{ and } P_{\mathrm{2D}}(r) = 2\pi\int_0^\infty E(q)J_0(2\pi qr)q\,\mathrm{d}q.$$

In Figure 3, we illustrate the propagator for a one-dimensional restricted geometry, where the effect of restrictions is characterized by a piecewise linear propagator. However, the same one-dimensional Fourier transform of the signal for spherical pores leads to a smooth bell-shaped curve, whereas the signature of restrictions becomes visible if the three-dimensional isotropic transform is applied on the MR signal profile.

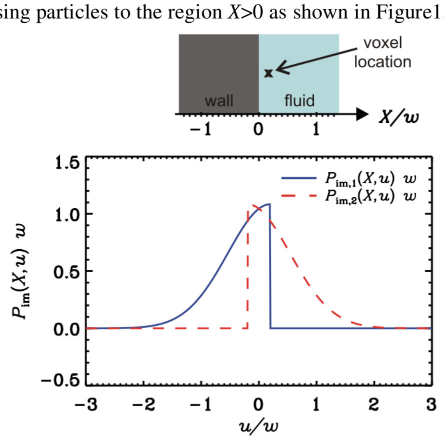

Fig. 1. Top: a sketch of the partially restricted geometry. Bottom: the propagators obtained in the traditional and new ways.

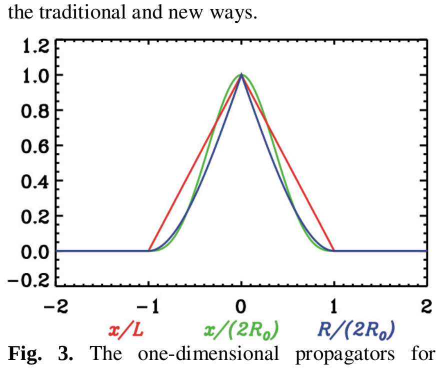

Fig. 3. The one-dimensional propagators for diffusion taking place between two parallel plates separated by L (red) and inside a spherical pore of radius R_0 (green). The blue curve is produced by employing the three-dimensional isotropic transform to the signal for the spherical pore.

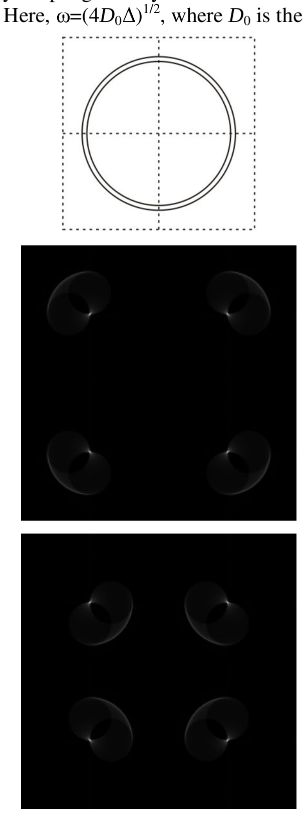

Fig. 2. Top: the circular geometry.Each quadrant of the circle is within one of the voxels of the four-voxel image. The propagators derived in the traditional (middle) and new (bottom) ways.

DISCUSSION & CONCLUSION

Non-symmetric diffusion propagators can be mapped from the complex-valued MRI signal [5]. When this is done, one should exercise caution about the direction of the Fourier transform and its associated meaning as one may lead to a more intuitive outcome than the other. In axially-symmetric and isotropic environments transforms other than the one-dimensional Fourier transform can be employed, which may make information available otherwise obscured by the Fourier transform.

References: [1] Y. Cohen, Y. Assaf, NMR Biomed, 15, 516-42 (2002). [2] V. J. Wedeen et al., Magn Reson Med, 54, 1377-86, (2005). [3] E. Ozarslan et al., Biophys J, 94, 2809-18 (2008). [4] M. A. Stephens, Biometrika, 50, 385-90 (1963). [5] C. L. Liu et al., Isr J Chem, 43, 145-54 (2003).