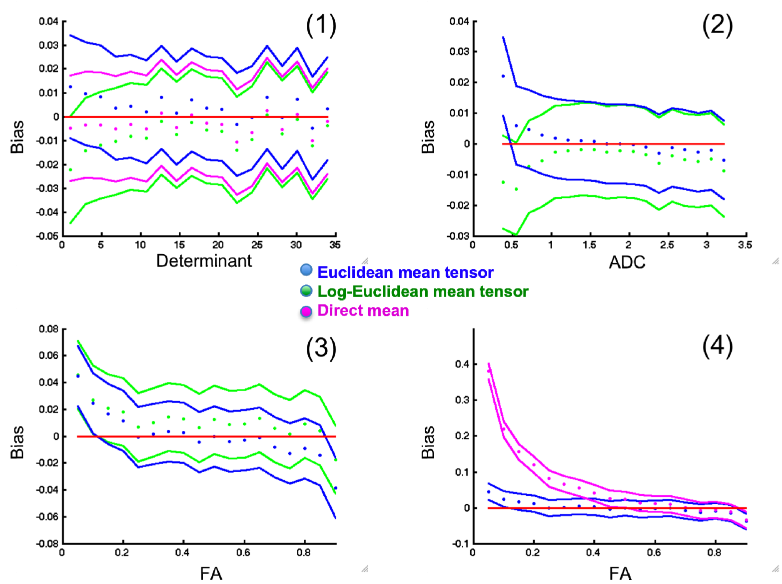
The Effect of Metric Selection on Averaging Diffusion Tensors – When and Why do Tensors Swell?

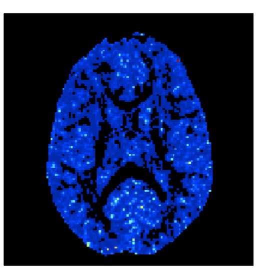
O. Pasternak[1], N. Sochen[2], and P. J. Basser[3]
[1]Brigham and Women's Hosptial, Harvard Medical School, Boston, MA, United States, [2]Tel Aviv University, Israel, [3]Section on Tissue Biophysics & Biomimetics (STBB), National Institutes of Health (NIH), Bethesda, MD, United States

Introduction In many diffusion tensor imaging (DTI) analysis methods, including registration, realignment and re-slicing, averaging or interpolating tensors is required. Defining how distance is measured between tensors, through a metric, determines the interpolation results. It was demonstrated that when using a conventional Euclidean metric, the resulted tensor might have a larger volume (determinant) than the original tensors [1]. This effect was termed the “swelling effect” [2] and a family of geometric metrics that include the Affine-invariant metric [3] and the Log-Euclidean metric [1] were suggested in order to minimize it. However, recently it was shown that using the geometric metrics introduces bias in the estimation of the diffusion quantities, which renders these metrics inappropriate for diffusion tensor analysis [4]. In this work we seek to find the sources of the swelling effect by performing tensor averaging using a Euclidean metric and a Log-Euclidean metric, and observing the bias in estimating FA, ADC and volume. We show that (i) using the Log-Euclidean metric reduces swelling, but introduces other types of biases in the estimation of ADC and FA; (ii) depending on the type of noise, a swollen tensor may be a preferred estimate. We argue that unwanted swelling effect is limited to certain scenarios, yet neither metrics help in avoiding it then.

Methods Data generation follows the synthetic experiments reported in [4]: noisy replications of tensor images were generated by selecting a reference tensor image (a single slice taken from a DTI acquisition of a healthy volunteer) and introducing Johnson noise (Rician distributed) using Monte-Carlo simulations to reproduce noisy replicates. Tensor fields were fitted to each replicate using the fitting procedure in [5] that assures positive definite tensors. Then, for each voxel a mean tensor was calculated, either using the Euclidean metric (arithmetic mean) or the Log-Euclidean metric (geometric mean). Maps of FA, ADC and determinant were generated for both types of mean tensor. In addition, “direct mean” FA, ADC and determinant maps were calculated by simply averaging the individual maps calculated for each noisy replicate. Voxel-wise bias, bias = $(E(x) - x_0)/x_0$, was calculated by comparing each average map ($E(x)$) with the FA, ADC and determinant maps of the reference image (x_0). The values shown in the plots are the mean bias (dots) and standard deviation (solid lines) across all voxels in the quantity (ADC, FA or determinant) range.



Results and Discussion
Figure (1) shows the bias in estimating the determinant, or volume of the tensors. Figure (2) shows the bias in estimating ADC and figures (3-4) show the bias in estimating FA. Positive bias suggests overestimation while negative suggests underestimation. The Euclidean metric consistently over estimates the determinant, over estimates lower ADC values and lower FA values, and under estimates higher ADC values and higher FA values. This indeed suggests a tendency to swell (higher determinant, lower FA). The Log-Euclidean metric consistently under estimates the determinant, which suggest shrinking, yet at the cost of consistently under estimating ADC, and over estimating most FA values. The size of the bias for both metrics is comparable, yet in FA estimations is consistently lower for the Euclidean metric relative to the Log-Euclidean metric. Estimating the direct mean ADC is equivalent to estimating the ADC of the Euclidean mean tensor (since these are linear operators), yet the effect of using the direct mean estimate is evident in the FA estimation bias shown in figure (4). The direct estimation introduces high bias in the form of over-estimation when estimating low FA voxels. This effect is explained since noisy replicates of initially low FA tensors are likely to have a higher FA. Direct estimating the mean FA of all replicates is then an average of overestimated values and is overestimated by it self. The swelling effect of the Euclidean metric is beneficial here, since the mean tensor had lower determinant and FA than the noisy tensor replicates and, in fact, better resembles the original tensor, yielding much lower bias. The intensity of figure (5) is proportional to the difference between the direct mean FA estimation bias and the Euclidean mean tensor FA estimation bias, showing that indeed the direct approach over estimates FA, mainly in isotropic voxels (gray matter and CSF). Unwanted swelling effect is therefore limited mainly to white-matter voxels and may only be evident when the source of variability affects mainly the orientation (unlike Johnson noise). This may be the case in miss-registration across repeated scans, or across subjects, that result with the same fiber tract oriented differently. For this special case we argue that a global metric (either the Euclidean or the Log-Eucldean) is not helpful, and local, tissue specific metrics that operate on subset of tensors are required.



(5)

Summary. The findings of this experiment are that the Euclidean and Log-Euclidean metrics show similar behavior in the estimation of ADC and FA for the range of tensors that are expected to occur in DTI of the brain. The Euclidean metric is consistently less biased in the estimation of FA. These findings together with previously published theoretical and statistical analysis of metric selection effects [4,6] provide empirical reasons for preferring the Euclidean metric over the tested geometric metrics.

References:[1] Arsigny et al., MRM 56, 2006. [2] Batchelor et al., MRM 53, 2005 [3] Pennec, JMIV 25, 2006. [4] Pasternak et al., Neuroimage, in print [5] Fillard et al., IEEE-TMI 26, 2007. [6] Whitcher et al., MRM 57, 2007.