Diffusion Tensor Image Registration Using Uncertainty Information

M. O. Irfanoglu[1,2], C. G. Koay[3], S. Pajevic[3], R. Machiraju[4], and P. J. Basser[3]

[1]Computer Sciences and Engineering, The Ohio State University, Columbus, OH, United States, [2]NICHD, NIH, Bethesda, MD, United States, [3]NIH, [4]The Ohio State University

Introduction: Population and longitudinal analyses using Diffusion Tensor Imaging (DTI) data have become feasible over the past decade with advanced sequences and sophisticated mathematical tools. These studies make use of some form of elastic tensor field registration framework to derive a population average brain and the deviation modes. These registration algorithms need to employ a specialized tensor similarity metric [1,2] and an tensor interpolation method [3]. Previously proposed metrics directly use tensor-derived information and disregard the diffusion weighted data once tensor fitting has been performed. In this work, we propose a new, analytical tensor similarity metric that not only inherently considers tensor directionality and shape but that also uses the entire experimental design information to infer the uncertainty in the computed tensors.

Materials and Methods:

Tensor covariance: In this work, we do not consider diffusion tensors as deterministic quantities but rather random variables, which are functions of another set of random variables, i.e. the diffusion weighted measurements. The function is the non-linear tensor fitting function, which can be expressed as: $f_{NLS}(\gamma) = \frac{1}{2}\sum_{i=1}^{n}\left(s_i - \exp\left[\sum_{i=1}^{7}\mathbf{W}_{ij}\gamma_j\right]\right)^2$ where γ represents the vectorized diffusion tensor, s_i is the measured diffusion signal corrupted with noise and $\mathbf{W}$ the experimental design matrix as in [4]. The shape of this function, its Hessian matrix, at the optimum solution $\hat{\gamma}$ is an indicator of the robustness of the computed diffusion tensor where, a flat shape indicates that a large range of diffusion tensors yield the same optimum function value. Koay et al. [4] showed using error propagation techniques that the invariant Hessian of this function can be computed with $\nabla^2 f_{NLS}(\gamma) = \mathbf{W}^T(\hat{\mathbf{S}}^2 - \mathbf{R}\hat{\mathbf{S}})\mathbf{W}$ where S and $\hat{S}$ are diagonal matrices of observed and estimated signals respectively and R=S -$\hat{S}$. The variance covariance matrix of the diffusion tensor components can then be computed as $\mathbf{\Sigma}_\gamma = \sigma_{DW}^2\left[\nabla^2 f_{NLS}(\hat{\gamma})\right]^{-1}$ with σ^2_{DW} representing the noise in DWIs.

Tensor similarity metric: With the diffusion tensors regarded as normally distributed random variables, one can employ a distribution similarity function as tensor similarity metric. We model the overall tensor field similarity metric F between a fixed DTI image I_f and a moving DTI image I_m:

$$F(I_f, I_m, \Theta) = \frac{1}{N}\sum_{p\in\Omega} w_p(I_f, I_m)\begin{pmatrix} tr(\mathbf{\Sigma}_m^{-1}\mathbf{\Sigma}_f) + (\gamma_m' - \gamma_f)^T\mathbf{\Sigma}_m^{-1}(\gamma_m' - \gamma_f) \\ + \\ tr(\mathbf{\Sigma}_f^{-1}\mathbf{\Sigma}_m) + (\gamma_f - \gamma_m')^T\mathbf{\Sigma}_f^{-1}(\gamma_f - \gamma_m') \end{pmatrix} \quad (1)$$

In Equation 1, p represents the voxel indices, Ω the image domain, γ_f signifies γ_f(p), the diffusion tensor on the fixed image at location p; similarly Σ_f is Σ_f (p) the tensor covariance matrix at the same location, and Θ the transformation parameters. For the moving image the variables represent the interpolated versions, i.e. Σ_m corresponds to Σ_m(T(p, Θ)), where the interpolation of the matrices was performed through a continuous B-splines approximation [5]. w_p represents a voxel error weighting function based on the tissue type.

Dimensionality Reduction. With a 6 dimensional vector representing the independent diffusion tensor components and 21 dimensions for the 6x6 tensor covariance matrix, this approach suffers from curse of dimensionality and slow convergence speed. However, one can note that the form of the covariance matrix depends on the experimental design. As shown in [6], with dense sampling of the unit icosahedrons with diffusion gradients, this covariance matrix should converge to the isotropic design matrix form of Table 1with 2 variables. Therefore, the full diffusion tensor covariance matrices are projected onto this form and the variables λ and μ derive the distribution.

$$\begin{pmatrix} \lambda+2\mu & \lambda & \lambda & 0 & 0 & 0 \\ \lambda & \lambda+2\mu & \lambda & 0 & 0 & 0 \\ \lambda & \lambda & \lambda+2\mu & 0 & 0 & 0 \\ 0 & 0 & 0 & 4\mu & 0 & 0 \\ 0 & 0 & 0 & 0 & 4\mu & 0 \\ 0 & 0 & 0 & 0 & 0 & 4\mu \end{pmatrix}$$

Table 1. Isotropic design matrix

Data and Experiments. Data from six healthy subjects were acquired with 10 b=0 s/mm^2 and 62 b=1000s/mm^2 volumes. Matrix sizes for all images were 128x157 with 114 axial slices and 1.5mm^3 isotropic voxel resolution. Five DT images from the dataset were registered to the first image with rigid transform, followed by affine transform and deformable B-Splines transform with a 20x20x20 grid size. Voxelwise standard deviations of tensor derived quantities were employed to assess registration quality.

Results: Figure 1 displays the output of the registration algorithm. The metric proves to perform well on white matter regions, as can be observed from the similarity of the images in the first and third columns. The difference image of the registered moving image and the fixed image is displayed on the fourth column. This difference image does not contain any particular structure or tissue dependent characteristics. A similar behavior can be observed in Figure 2, which displays the voxelwise standard deviation of FA values over six subjects.

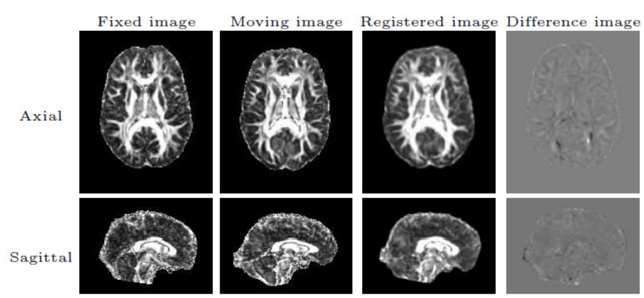

Figure 1. Registration outputs. Fixed DT image is registered to the moving image. The output of the registration is displayed in the 3rd column. The FA maps are computed after the tensor registration process.

Discussions: In this work, we proposed a diffusion tensor similarity metric for tensor field registration for population or longitudinal analysis. This metric is particularly suitable for situations with high uncertainty including images with low SNR. Additionally, for DTI data acquired with different number of gradients or different gradient orientations, such as multi-center population studies proposed metric provides a convenient approach to deal with the bias introduced the to the statistics by the experimental design.

The proposed metric outputs a larger value for the same set of two diffusion tensors with increasing certainty and the optimization procedure employed by registration prefers to minimize the errors in these high certainty regions over uncertainty regions. This is an important property for images with spatially varying SNR and as a future work, we will investigate this behavior with synthetic phantoms and with images acquired with parallel imaging.

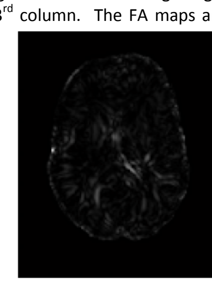

Figure 2. Voxelwise std of FA over six subjects.

References: 1. Zhang H et al. TMI, 2007; 2.Cao Y et al., CVPR, 2006; 3. Arsigny V et al, MRM 2006; 4. Koay CG et al., TMI, 2007; 5. Pajevic S et al, 2006; 6. Basser PJ et al, TMI, 2003.