Accurate Estimation of T1 from SPGR Signals

L-C. Chang[1,2], C. Koay[2], P. J. Basser[2], and C. Pierpaoli[2]

[1]Department of Electrical Engineering & Computer Science, The Catholic University of America, Washington, DC, United States, [2]NICHD, National Institutes of Health, Bethesda, Maryland, United States

Introduction

T_1 maps can be computed from spoiled gradient recalled echo (SPGR) images acquired with different flip angles and/or repetition times (*TRs*). The function relating signal intensity to flip angle and *TR* is non-linear; however, a linear form proposed by Gupta in 1977 [1] is currently widely used [1-6]. Using this linearized model, T_1 has been estimated with a linear least squares (LLS) method, which has the advantage of being computationally efficient. However, our preliminary study found that the estimated T_1 using this LLS method was generally biased and over-estimated [7]. We propose a new weighted linear least squares (WLLS) approach that uses adjusted uncertainties in the fitting. The proposed WLLS method weights each data point with the uncertainty that corrects the noise contribution produced by the transformation of a nonlinear model to a linear one. Numerical and human brain data simulations are used to compare the accuracy of T_1 estimated using the LLS, WLLS, and nonlinear least squares (NLS) methods.

Theory

The measured SPGR signal intensity can be written as equation (1), where α_i is the flip angle, M_0 is the equilibrium longitudinal magnetization, and $E_1 = \exp(-TR/T_1)$ [1, 2]. The NLS method estimates T_1 and M_0 from equation (1) by minimizing the objective function (2). Linear fitting can be used if all images in the dataset are collected with the same *TR*; equation (1) can be represented in linear form as equation (3). The LLS method estimates T_1 and M_0 from equation (3) by minimizing the objective function (4), where $y_i = s_i / \sin(\alpha_i)$, $x_i = s_i / \tan(\alpha_i)$, $b = E_1$, and $a = M_0(1-E_1)$. The proposed WLLS method also estimates T_1 and M_0 from equation (3) by minimizing another objective function (5), where y_i, x_i, a, and b have the same definition as in the LLS method. The weighting function in equation (5) can be derived directly from equation (2) or by using error propagation analysis [8].

$$s_i = \frac{M_0(1-E_1)\sin(\alpha_i)}{1-E_1\cos(\alpha_i)} \qquad (1)$$

$$\chi^2_{NLS}(M_0,\ T_1) = \sum_{i=1}^{n} \frac{1}{\sigma_i^2}\left(s_i - \frac{M_0(1-E_1)\sin(\alpha_i)}{1-E_1\cos(\alpha_i)}\right)^2 \qquad (2)$$

$$\frac{s_i}{\sin(\alpha_i)} = E_1 \frac{s_i}{\tan(\alpha_i)} + M_0(1-E_1) \qquad (3)$$

$$\chi^2_{LLS}(a,b) = \sum_{i=1}^{n} \frac{1}{\sigma_i^2}\left(y_i - a - bx_i\right)^2 \qquad (4)$$

$$\chi^2_{WLLS}(a,b) = \sum_{i=1}^{n} w_i\left(y_i - a - bx_i\right)^2 \qquad (5)$$

$$w_i = \frac{1}{\sigma_i^2}\left(\frac{\sin(\alpha_i)}{1-E_1\cos(\alpha_i)}\right)^2 \qquad (6)$$

Simulations

We performed simulations of different experimental designs, and different expected values of T_1. Different signal to noise (SNR_0), expressed as M_0/σ, were simulated by adding (in quadrature) Gaussian noise with zero mean and variable standard deviation, σ, to the noise-free SPGR signals generated using equation (1). Results reported below are computed assuming *TR*=10ms and M_0=3000, optimal flip angles were computed based on the true T_1 as suggested in [3]. Six SPGR images, consisting of three replicates of two flip angles without averaging, were used. The accuracy of the estimated T_1 was also tested on synthetic data derived from T1 measurements in the human brain. The strategy used to create the synthetic human brain data was similar to that described in [9]. The resultant SPGR brain images have $SNR_0 = M_0/\sigma$ ranging from 90-150 throughout most areas of the brain tissue.

Results

Fig. 1 shows the estimated T_1 using the LLS, WLLS, and NLS methods for SNR_0 ranging from 30 to 300. WLLS and NLS produce estimates of T_1 with comparable accuracy at all SNR_0 tested, while LLS overestimates T_1 progressively as SNR_0 decreases. Fig. 2 shows the relative error of T_1 using the LLS, WLLS, and NLS methods for T_1 ranging from 600 to 2000 ms with a fixed SNR_0 = 100. The relative error is defined as (true T_1 – estimated T_1)/true T_1. The bias of T_1 is corrected in WLLS with the relative error less than 5% regardless of the value of T_1. Fig 3 shows maps of the relative error on the estimated T_1 in the synthetic human brain data using the LLS and WLLS methods. The results shown in Fig. 3 (b) and (c) were scaled in the range of ± 20%, the gray background corresponds to zero. The relative error of LLS is consistently higher than that of WLLS in brain tissue, and is positive, indicating that T_1 is overestimated.

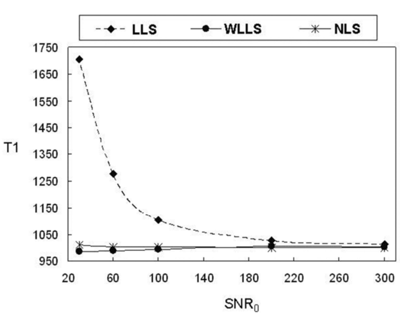

Fig. 1 Estimated T_1 using the LLS, WLLC, and NLS methods assuming a true T_1 value of 1000ms.

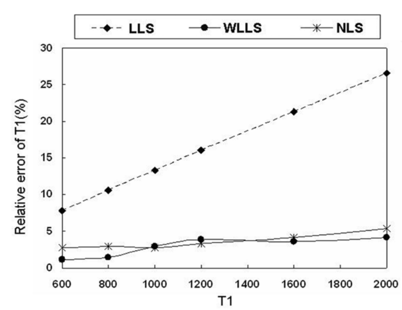

Fig. 2 Relative error of T_1 using the LLS, WLLS, and NLS methods with SNR_0 = 100.

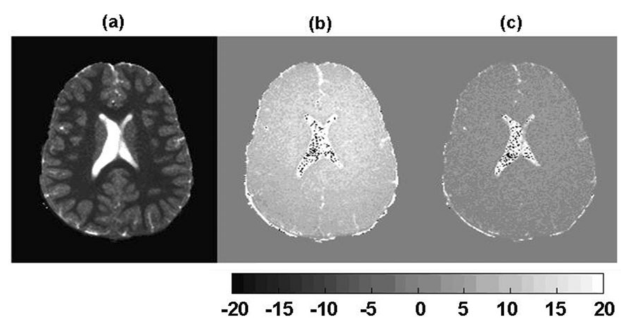

Fig. 3 Relative error of T_1 on a selected slice of synthetic human brain data using (b) LLS and (c) WLLS methods in the fitting procedure. The true T_1-map of the same slice is shown in (a) for reference.

Discussion & Conclusion

In this work we show that the widely-used LLS method improperly weights the uncertainty, resulting in significant errors in T_1 estimation. For T_1 values ranging from 800 to 1600 ms, which cover most of the T_1 values in brain tissue, T_1 is overestimated by 10-20%.We propose a weighting approach for the linear model that uses properly weighted uncertainties to adjust the noise contribution produced by the linear transformation. The proposed weighted linear least squares method yields estimated T_1 with a precision and accuracy comparable to that obtained from nonlinear fitting while reducing the computation time significantly, enabling the generation of accurate T_1 maps "on the fly" at the scanner console.

References:[1] Gupta, *J Magn Reson* 25:231-235, 1977. [2] Deoni et al, *Magn Reson Med* 49: 515-526, 2003. [3] Wang et al, *Magn Reson Med* 5: 399-416, 1987. [4] Deoni et al, *Magn Reson Med* 51: 194-199, 2004. [5] Fram et al, *Magn Reson Imaging* 5(3):201-208, 1987. [6] Cheng et al, *Magn Reson Med* 55(3):566-574, 2006. [7] Chang et al, the 15th Annual Meeting of ISMRM: 1775, Berlin, Germany 2007. [8] Bevington. McGraw-Hill Book Company, New York, NY 1969. [9] Chang et al, *Magn Reson Med* 53(5):1088-1095, 2005.