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Anisotropically Weighted MRI

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INTRODUCTION

Diffusion tensor MRI (1) is proving useful in elucidating microstructural features of healthy and diseased tissues. For example, MRIs whose contrast is a function of Trace($\underline{D}$) (where $\underline{D}$ is the diffusion tensor), have been used in the diagnosis of acute stroke (2). Such isotropically-weighted MRIs have been calculated recently using as few as two diffusion weighted images (DWIs) (3-5). Robert Turner recently asked whether one could construct an "anisotropically weighted" MRI sequence (6) (i.e., one whose contrast depends on a quantitative measure of diffusion anisotropy) that also uses a small number of DWIs. This development would be relevant clinically because maps of diffusion anisotropy reflect tissue microarchitecture and its changes in disease, such in Wallerian degeneration and organized gliosis observed in chronic stroke patients (7). Here we (a) determine the fewest number of DWIs required to construct an anisotropically weighted image, (b) outline a method to construct anisotropically weighted MRIs from DWIs, and (c) discuss the relative merits of diffusion tensor MRI and anisotropically weighted MRI:

THEORY

A number of admissible quantitative measures of diffusion anisotropy are functions of the second or higher moments of the three eigenvalues (principal diffusivities) of $\underline{D}$ in each voxel, where the n^{th} moment is proportional to:

$$(\lambda_1 - <\lambda>)^n + (\lambda_2 - <\lambda>)^n + (\lambda_3 - <\lambda>)^n$$

and

$$<\lambda> = <D> = \mathrm{Trace}(\underline{D})/3 .$$

Here we outline a procedure to calculate an anisotropically weighted image whose intensity is proportional to a moment of the eigenvalues of $\underline{D}$.

To do this, first we show that the n^{th} moment of the eigenvalues of $\underline{D}$ can be written as an n^{th} order polynomial whose terms are all products of elements of $\underline{D}$ (e.g., $D_{xx}{}^n$, $D_{xy}{}^{n-2} D_{yz} D_{xz}$, ...). Next, using the fundamental equation of diffusion tensor imaging (8), we show that the logarithm of the attenuation of the echo amplitude, $\ln(A(b)/A(0))$, raised to the n^{th} power also can be written as an n^{th} order polynomial whose terms are likewise products of elements of $\underline{D}$. This is the case for products of different log attenuation images, e.g., produced from two or more DWIs, such as $(\ln(A_1(b)/A(0)))^{n-1}$ $(\ln(A_2(b)/A(0)))$ ...), whose degree is n.

To determine how many independent DWIs are required to compute an image whose intensity is proportional to the n^{th} moment of the eigenvalues of $\underline{D}$, we form the most general n^{th} order polynomial from a set of DWIs in which each term is of degree n. We then try to find a set of polynomial coefficients so that the linear combination of products of log attenuation images equals the polynomial expression of the n^{th} moment of the eigenvalues of $\underline{D}$.

This can be reduced to a familiar problem in linear algebra--determining whether a system of linear equations has 0, 1, or more solutions. For each value of n, and for a specified number of DWIs, we can determine the relationship between the column space spanned by the n^{th} moment of the eigenvalues of $\underline{D}$ and the column space spanned by the most general linear combination of products of the log attenuation images formed from the set of DWIs.

RESULTS

First, we show that the smaller the moment, the fewer DWIs are required to calculate it. Therefore, if we determine the minimum number of DWIs required to compute the second moment, then this will provide a lower bound for all higher moments. Next, we show using theorems of nullity and rank that for six or fewer DWIs, it is algebraically impossible to represent the second moment using sums of products of log attenuation images. *Seven is the minimum number of DWIs required to produce an anisotropically weighted image.*

Another interesting result is that while it is possible to sensitize a DWI exclusively to the isotropic part of $\underline{D}$ without also producing some attenuation due to anisotropic diffusion, it is impossible to sensitize the DWI exclusively to the anisotropic part (or deviatoric) of $\underline{D}$ (9) without also producing some attenuation caused by isotropic diffusion.

DISCUSSION

Our results are quite general. They are largely independent of the details of the experimental design, e.g., the gradient magnitudes, directions, timing parameters, etc. For example, almost any set of seven DWIs with gradients applied in oblique directions can be used to construct an image of the second moment of the eigenvalues of $\underline{D}$, although other considerations (e.g., tissue properties, SNR, etc.) inevitably will make some combinations of DWIs preferable to others.

CONCLUDING REMARKS

Here we outlined a procedure to calculate an anisotropically weighted image, and a proof that at least seven DWIs are required to produce one. Seven is also the minimum number of DWIs required to estimate $\underline{D}$ directly as well as the T2-weighted amplitude image, A(0) in each voxel. On the basis of information gain per image, ease of implementation, and noise immunity, diffusion tensor MRI appears to be more general and easier to implement than anisotropically-weighted MRI.

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