

PROCEEDINGS

of the

International Society for Magnetic Resonance in Medicine

FIFTH SCIENTIFIC
MEETING AND EXHIBITION

Vancouver, B.C., Canada
• April 12-18, 1997

Volume 1

Analytic Expressions for the Diffusion Tensor Elements

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INTRODUCTION

In related studies, we have demonstrated that at least seven diffusion weighted images (DWIs) are required to characterize diffusion anisotropy in tissues adequately (1). However, patient welfare and cost considerations behoove us to minimize the number of DWIs we acquire. Therefore, it is prudent to assess the feasibility and merits of performing diffusion tensor MRI using only seven DWIs.

METHODS

Using a 2-T CSI animal imaging system and a birdcage quadrature coil, 25 (128x128) axial, 4-slice, DW, 2D-FT echo-planar images of live anesthetized cat brain were acquired, each in 96 seconds. Diffusion gradients were applied in 6 oblique directions (2): $\{(1,0,1), (-1,0,1), (0,1,1), (0,1,-1), (1,1,0), (-1,1,0)\}$ using an interleaved, navigator echo-corrected sequence (3, 4) with four different diffusion weightings: $\text{Trace}(\underline{b}) = 229, 461, 693,$ and 915 sec/mm^2 , where \underline{b} is the b-matrix. An additional DWI was acquired with effectively no diffusion weighting ($A(\underline{b}=0)$). All 25 DWIs per slice were acquired with the following parameters: $TE=75.6\text{ms}$, $\Delta=38.9\text{ms}$, $\delta=14\text{ms}$, $TR=3\text{sec}$, $na=2$, $FOV=80\text{mm}$, $st=2.5\text{mm}$. For each slice, \underline{D} was calculated in each voxel for each set of six DWIs with the same diffusion weighting (i.e., $\text{Trace}(\underline{b})$), and $A(\underline{b}=0)$:

$$D_{xz} = \frac{-IM_1 + IM_2}{4b}; D_{yz} = \frac{-IM_3 + IM_4}{4b}; D_{xy} = \frac{-IM_5 + IM_6}{4b};$$

$$D_{zz} = \frac{-IM_1 - IM_2 - IM_3 - IM_4 + IM_5 + IM_6}{4b};$$

$$D_{xx} = \frac{-IM_1 - IM_2 + IM_3 + IM_4 - IM_5 - IM_6}{4b}; \text{ and}$$

$$D_{yy} = \frac{IM_1 + IM_2 - IM_3 - IM_4 - IM_5 - IM_6}{4b},$$

where $b = 1/2 \alpha^2 G_0^2$ (5) and $IM_i = \text{Log}(A_i(\underline{b})/A(\underline{b}=0))$. The entire set of 25 DWIs per slice and their corresponding b-matrices (6) were also used to estimate \underline{D} and $A(\underline{b}=0)$ in each voxel (5). For comparison, maps of $\text{Trace}(\underline{D})$ (7) and the relative anisotropy (8) were calculated both from the analytical expressions for \underline{D} above, and the statistically estimated \underline{D} (5).

RESULTS

In Figures 1(a,b) we see surprisingly good quality maps of the relative anisotropy (RA) and $\text{Trace}(\underline{D})$ calculated analytically using DWIs acquired with the highest diffusion weighting ($\text{Trace}(\underline{b}) \sim 915 \text{ sec/mm}^2$). Good agreement is achieved with statistically estimated maps (not shown). However, the quality of the analytically computed maps degrades markedly when using lower diffusion weightings, characteristic of most clinical imagers.

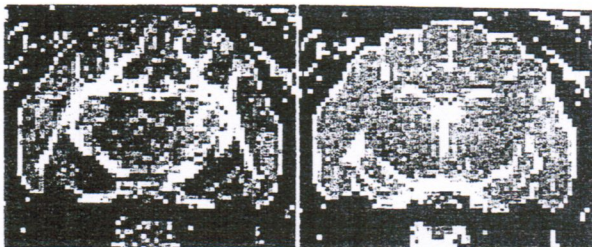


Fig 1: (a) RA and (b) $\text{Trace}(\underline{D})$ maps

DISCUSSION

The success of this simplified DT-MRI scheme depends on several conditions being satisfied. (1.) To obtain such simple algebraic expressions for \underline{D} (and for its invariants) the contributions of all imaging gradients (cross-terms) on the signal attenuation are neglected. Our sequence was designed specifically to satisfy this condition. (2.) The DWIs are all free of noise. Therefore, as SNR decreases, the quality of the computed maps (e.g., of $\text{Trace}(\underline{D})$ and diffusion anisotropy) degrades. (3.) The dynamic range of diffusion weighting should be adequate. For example, to estimate the mean diffusivity, $\langle D \rangle$, one should choose $\text{Trace}(\underline{b}) \sim 1/\langle D \rangle$. For brain parenchyma, $\langle D \rangle \sim 700 \times 10^{-6} \text{ mm}^2/\text{sec}$ (9), so $\text{Trace}(\underline{b}) \sim 1430 \text{ sec/mm}^2$, which is significantly larger than most clinical imagers can achieve. (4.) DWIs should be free of systematic artifacts, in particular (a) ghosting caused by motion, (b) background gradients or magnetic susceptibility variations, (c) induced eddy-currents, and (d) improperly calibrated gradients. In a clinic, it likely that several DWIs will be corrupted by one or more of these artifacts.

The difficulty to satisfy these requirements in a clinical setting suggests why we do not presently recommend 7-DWI DT-MRI for routine radiological assessments. However, it still holds promise for the following applications: (1.) The first seven DWIs can be used to provide preliminary maps of $\text{Trace}(\underline{D})$ (or $\langle D \rangle$), of diffusion anisotropy measures, the principal diffusivities and principal axes, e.g., to help identify pathological regions. Still, if possible, additional DWIs should be acquired. (2.) In an experiment in which \underline{D} is expected to change with time, one can use 7 contiguous DWIs to obtain analytical expressions for \underline{D} at different time points (Pierpaoli et al. used a similar approach to follow the kinetic behavior of $\text{Trace}(\underline{D})$ and the principal diffusivities in acute ischemia (10)). (3.) The first seven DWIs can also furnish an initial estimate of \underline{D} and of $A(\underline{b}=0)$ in an iterative or recursive scheme in which each additional DWI is used to update these initial estimates.

CONCLUSIONS

In the special case of diffusion tensor MRI using seven DWIs, we can derive analytical expressions for the diffusion tensor of water, \underline{D} , and of its scalar invariants, explicitly in terms of the intensities of the seven DWIs. These formulae subsume the post-processing steps of (a) calculating a b-matrix for each DWI and (b) estimating \underline{D} in each voxel from the set of b-matrices and their corresponding DWIs. However, because of the susceptibility of this simplified method to experimental artifacts, it is not recommended for a radiological assessment. Nonetheless, it can provide useful information to help arrive at one.

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