A SENSITIVE METHOD TO CALIBRATE MAGNETIC FIELD GRADIENTS USING THE DIFFUSION TENSOR

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INTRODUCTION

A conventional method of calibrating magnetic field gradients is to acquire MRIs of a phantom of known shape, and iteratively adjust the gradient strengths until the image appears geometrically similar to the known phantom. From the imaging equation, the image's dimension in a particular direction is inversely proportional to the magnitude of the gradient strength in that direction, $|\mathbf{G}|$. However, applications such as diffusion spectroscopy (1) and imaging (2), diffusion *tensor* spectroscopy and imaging (3), MR microimaging, and MRI guided-surgery, all require more precisely calibrated magnetic field gradients than one can obtain by phase corrections. In the latter case a high precision is required for spatial localization, whereas in the former three cases, large gradients are applied whose strengths must be known precisely. New MR methods that require stronger or more accurate magnetic field gradients require sensitive methods to calibrate them.

THEORY

The scalar diffusion coefficient in a particular direction, estimated from an MR diffusion spectroscopy sequence, is inversely proportional to the *square* of the applied diffusion gradient strength in that direction, $|\mathbf{G}|^2$ (4). Similarly, in diffusion tensor spectroscopy, each element of the diffusion tensor, D_{ij}, is inversely proportional to the product of components of the diffusion gradients, $G_i\, G_j$ (5). Therefore, the percentage error in the estimated diffusion coefficient (6) (or diffusion tensor element) is twice the percentage error in the image size, for the same percentage error in gradient strength.

We propose a new method of calibrating magnetic field gradients that exploits the high sensitivity of diffusion attenuation to the magnetic field gradient strength. It is predicated on the premise that an isotropic medium (e.g., water) has an isotropic effective diffusion tensor, $\underline{\underline{D}}^\circ$, i.e.,

$$\underline{\underline{D}}^\circ = D_0\, \underline{\underline{I}} \quad [1]$$

(D_0 is the scalar self-diffusion constant; $\underline{\underline{I}}$ is the identity matrix or tensor). If the observed anisotropy in $\underline{\underline{D}}$ is ascribed to erroneous calibration of, cross-talk between, or misalignment of the x, y, and z magnetic field gradients, then we can correct for all three artifacts simultaneously by finding a transformation between the prescribed and actual magnetic field gradients that makes the measured diffusion tensor, $\underline{\underline{D}}$, of an isotropic medium appear isotropic. Analytically, this transformation is similar to an inverse filter (7); geometrically, it is tantamount to transforming a measured diffusion ellipsoid into a sphere.

For a PGSE diffusion sequence, the measured MR echo, A(TE), is related to the "true" effective diffusion tensor, $\underline{\underline{D}}^\circ$ using (5, 8):

$$\ln\left(\frac{A(TE)}{A(0)}\right) = -\alpha\ \mathbf{G}^{\circ T}\, \underline{\underline{D}}^\circ\, \mathbf{G}^\circ \quad , \quad [2]$$

where α is a known scalar that is calculated from gradient pulse timing parameters and the gyromagnetic ratio, and $\mathbf{G}^\circ = (G_x^\circ, G_y^\circ, G_z^\circ)^T$ is the "true" magnetic field gradient vector whose components are the peak applied field gradient amplitudes in the x-, y-, and z-directions. In diffusion tensor MR spectroscopy, we use Eq. [2], to estimate a measured diffusion tensor, $\underline{\underline{D}}$, from operator prescribed gradients, $\mathbf{G}$, that we assume are applied (5).

To obtain the relationship between the actual and ideal gradients, we equate their quadratic forms:

$$\mathbf{G}^{\circ T}\, \underline{\underline{D}}^\circ\, \mathbf{G}^\circ = (\underline{\underline{\Lambda}}^{1/2}\, \underline{\underline{E}}^T\, \mathbf{G})^T\, \underline{\underline{D}}^\circ\, (\underline{\underline{\Lambda}}^{1/2}\, \underline{\underline{E}}^T\, \mathbf{G})\ , \quad [3]$$

where we have diagonalized $\underline{\underline{D}}$ using its matrix of eigenvalues, $\underline{\underline{\Lambda}}$, and eigenvectors, $\underline{\underline{E}}$. Equating like terms above, we obtain

$$\mathbf{G}^\circ = \underline{\underline{\Lambda}}^{1/2}\, \underline{\underline{E}}^T\, \mathbf{G} \quad \text{or} \quad \underline{\underline{E}}\, \underline{\underline{\Lambda}}^{-1/2}\, \mathbf{G}^\circ = \mathbf{G} \quad [4]$$

We see that $\mathbf{G}$ is first rotated by $\underline{\underline{E}}^T$ and then stretched by $\underline{\underline{\Lambda}}^{1/2}$ to produce $\mathbf{G}^\circ$. To undo this distortion, we simply apply the inverse transformation, $\underline{\underline{E}}\, \underline{\underline{\Lambda}}^{-1/2}$ to $\mathbf{G}$.

DISCUSSION AND CONCLUDING REMARKS

Once we determine the form of the inverse filter matrix, we can correct $\underline{\underline{D}}$ measured in previous or subsequent experiments. While we can use any well-characterized anisotropic phantom, it is advantageous to use an isotropic phantom so that we do not need to specify its orientation with respect to the static magnetic field (or the laboratory coordinate system) as we must do with an anisotropic phantom. This is because the diffusion tensor of an isotropic medium is invariant -- it has the same form in *all* laboratory frames of reference, whereas the diffusion tensor of an anisotropic medium is not.

While we use diffusion spectroscopic sequences to calibrate the field gradients, we can also use diffusion imaging or diffusion tensor imaging sequences, provided that all contributions to the measured MR signal due to the imaging gradients are demonstrably negligible (9).

Caution should be exercised when using this method with either noisy or small data sets since the inverse filtering matrix may become singular. However, this requirement is easy to satisfy since we can obtain many high quality echoes rapidly (5). Care should also be exercised to remove sources of artifacts such as poor shimming, timing errors, susceptibility variations, and eddy currents.

If cross-talk between gradients or gradient misalignment exists, this method can be used to detect and eliminate them *at the time the gradients are manufactured or once they are installed on-site*. At the time of manufacture, errors can be corrected in hardware, whereas following installation errors can be corrected in software.

REFERENCES

1. Stejskal, EO, *J. Chem. Phys.*, 43, 3597-3603, 1965.
2. LeBihan, D et al., *Cr. Acad. Sci. (Paris)* , 301, 1109-1112, 1985.
3. Basser, PJ et al., *Biophys. J.*, 66, 259-267, 1994.
4. Stejskal, EO et al., *J.Chem.Phys.*, 42, 288, 1965.
5. Basser, PJ et al., *J Magn. Reson.*, B, 247-254, 1994.
6. Basser, PJ et al., in preparation.
7. Brigham, EO, *Fast Fourier Transform and its Applications*, 1988.
8. Basser, PJ et al., 11th SMRM, Berlin, 1, 1222, 1992.
9. Mattiello, J et al., *J. Magn. Reson.*, A 108, 131, 1994.