SOCIETY OF MAGNETIC RESONANCE IN MEDICINE

Book of Abstracts

Volume 1

Society of Magnetic Resonance in Medicine

Eleventh Annual Scientific Meeting
August 8 - 14, 1992
Berlin, Germany

held jointly with the
Ninth Annual Congress
of the
European Society for Magnetic Resonance
in Medicine and Biology

Fiber orientation mapping in an anisotropic medium with NMR diffusion spectroscopy

Peter J. Basser°, Denis LeBihan†
°Biomedical Engineering and Instrumentation Program, and
†Diagnostic Radiology Department, Warren G. Magnuson Clinical Center, NIH, Bethesda, MD 20892 USA

Purpose: The diagonal and off-diagonal components of the apparent self-diffusion tensor [1], $\underline{D}$, are used to construct a diffusion ellipsoid for a voxel that depicts both orientation of tissue microstructures, such as muscle fibers, and mean diffusion distances. New weighting parameters are also suggested for structural NMR imaging.

Principles: In heterogeneous, anisotropic media, $\underline{D}$ relates the flux of spin-labeled protons to their concentration gradient. Since $\underline{D}$ is symmetric and positive-definite, its three mutually orthogonal eigenvectors, ε_1, ε_2, and ε_3, define the principal axes and its three positive eigenvalues, λ_1, λ_2, and λ_3, are the diffusivities in these directions [2]. Continuum models of diffusion in heterogeneous media [3] suggest that the principal axes of $\underline{D}$ coincide with those of the grain or fiber and the principal diffusivities of $\underline{D}$ are related to the structure, geometry, and diffusivity of the various microscopic compartments within the heterogeneous medium.

The diffusion ellipsoid: In heterogeneous, anisotropic media [3], the macroscopic effective self-diffusion tensor, $\underline{D}$, appears in the conditional probability density function, which is the probability that a particle at x at time t was at x_0 at t = 0:

$$p(x\,|\,x_0,t) = \frac{1}{\sqrt{|\underline{D}|(4\pi t)^3}}\exp\left(\frac{-(x-x_0)^T\,\underline{D}^{-1}\,(x-x_0)}{4t}\right). \quad (1)$$

Setting the quadratic form to 1/2, i.e.,

$$\frac{(x-x_0)^T\,\underline{D}^{-1}\,(x-x_0)}{2t} = 1 \quad , \quad (2)$$

defines a diffusion ellipsoid whose principal axes constitute the local "fiber" frame of reference, and whose j^{th} major axis is the mean distance a spin-labeled proton diffuses in the j^{th} principal direction, $\sqrt{2\lambda_j\Delta}$, during the diffusion time, Δ.

The scalar invariants: Three scalar invariants of $\underline{D}$, I_1, I_2, and I_3, are:

$$\begin{aligned} I_1 &= \lambda_1 + \lambda_2 + \lambda_3 = \mathrm{Tr}\,\underline{D} \\ I_2 &= \lambda_1\lambda_2 + \lambda_3\lambda_1 + \lambda_2\lambda_3 \\ I_3 &= \lambda_1\lambda_2\lambda_3 = |\underline{D}| \;. \end{aligned} \quad (3)$$

They have the desirable properties of being independent of the coordinate system in which $\underline{D}$ is measured, and insensitive to the scheme by which the λ_i are numbered, making them (or functions of them), ideal weighting factors in structural NMR imaging.

Data analysis: Diffusion ellipsoids for pork loin are constructed from two apparent self-diffusion tensors, $\underline{D}^{0°}$ and $\underline{D}^{41°}$, estimated from spin-echo experiments [1]:

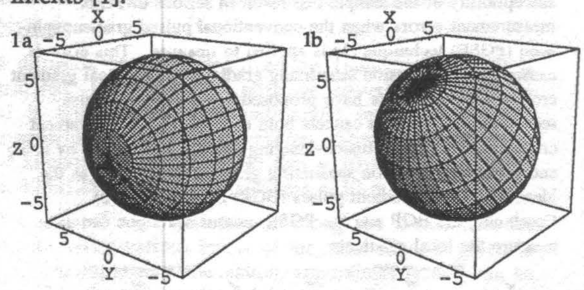

Fig. 1a,b: Diffusion ellipsoids of a pork loin sample. Laboratory coordinates, x, y, and z are in μm. The eigenvector corresponding to the largest eigenvalue defines the polar axis.

In Fig 1a, the grain of the sample was nearly aligned with the magnet's x axis. Eigenvalues (principal diffusivities) of $\underline{D}^{0°}$ are {λ_1 = (1.0406±0.0007)10^{-5}, λ_2 = (0.944±0.001)10^{-5}, λ_3 = (0.8532±0.0006)10^{-5}} (cm²/sec). In Fig. 1b, the same sample is rotated approximately 41° in the z-x plane. Eigenvalues of $\underline{D}^{41°}$ are {λ_1 = (1.0119±0.0003)10^{-5}, λ_2 = (0.9343±0.0006)10^{-5}, λ_3 = (0.8767±0.0018) 10^{-5}} (cm²/sec).

Discussion: The eigenvectors that define the fiber frame follow the sample when it is rotated. This is represented by the tipping of the polar axis. The scalar invariants of $\underline{D}$ differ by no more than 1% in both cases because they are intrinsic to $\underline{D}$, independent of the sample's orientation in the magnet. Both ellipsoids are nearly spherical, presumably because the diffusion time, Δ = 22.5 ms, corresponding to a mean diffusion distance of 4.7 μm, is too short for the majority of spin-labeled protons to encounter diffusional barriers.

Although a single voxel was used in this study, these principles can be generalized to multiple voxels. One could envision 3-D fiber maps [4] or diffusion ellipsoids displayed in each voxel, connected like link sausages that follow fiber tracts.

Conclusion: Constructing the diffusion ellipsoid requires knowledge of all diagonal and off-diagonal elements of $\underline{D}$. Inherently, $\underline{D}$ contains unique directional, structural and anatomical information *within a voxel* that scalars such as T_1 or T_2 do not - information that is embodied in the diffusion ellipsoid and scalar invariants.

References:

1. Basser, P. J., et al., submitted SMRM 1992.
2. Carslaw, H. S., Jaeger, J. C., Oxford Press, 1959.
3. Crank, J., Oxford Press, 1975.
4. Douek, P., et al., *J. Comput. Assist. Tomogr.* **15**, 923, 1991.