A Novel Tensor Distribution Model for the Diffusion Weighted MR Signal

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INTRODUCTION

We present a novel statistical model for diffusion-weighted MR signal attenuation which postulates that the local tissue geometry is characterized by a continuous mixture of diffusion tensors. An interesting observation is that the mixing distribution and the MR signal attenuation are related via a Laplace transform. We also show that when the mixing distribution is a Wishart distribution, the Laplace transform leads to a Rigaut-type function which has been phenomenologically used previously to explain the MR signal decay but never with a rigorous mathematical justification until now. We further develop a spherical deconvolution method for resolving multiple fiber orientations using the proposed model. Finally, we demonstrate that our technique is capable of generating several different quantities that are commonly computed using various other techniques to visualize the orientational preference of water diffusion.

THEORY

By extending the discrete Gaussian mixture model to the continuous case, we assume that each voxel is associated with a probability distribution F on P_n , the space of 3x3 positive definite matrices. The resulting model is as follows:

$$S(q)/S_0 = \int_{P_n} \exp(-bg^T Dg)\,dF = \int_{P_n} \exp(-trace\,(BD))\,dF$$

where $B = bgg^T$. An interesting observation is that the above model coincides with the definition of the Laplace transform for matrix valued distributions. Moreover, in the case of F being a Wishart distribution $\gamma_{p,\sigma}$ with scale parameter σ in P_n and shape parameter p (for the formal definition, see [1]), we obtain a closed form expression for the Laplace transform $S(q) = S_0(1+(bg^T\hat{D}g/p)^{-p}$ where $\hat{D} = p\sigma$ is the expected value of $\gamma_{p,\sigma}$. This is a familiar Rigaut-type asymptotic fractal expression [2] implying a signal decay characterized by a power-law which is the expected asymptotic behavior for the MR signal attenuation in porous media. Also note that when p tends to infinity, we have $S(q) \rightarrow S_0 \exp(-bg^T\hat{D}g)$ which suggests that the mono-exponential model can be viewed as a limiting case of our model.

METHODS

The Laplace transform relation between the MR signal and the probability distributions on P_n naturally gives rise to an inverse problem: to recover a distribution on P_n that best explains the observed diffusion signal. This is an ill-posed problem and in general is not solvable without further assumptions. We propose a discrete mixture of Wishart distribution model where the mixing distribution is expressed as $dF = \sum w_i d\gamma_{pi,\sigma i}$. We further assume that all components take the same value p=2 based on the analogy between the proposed model and the Debye-Porod law of diffraction [3]. Since the fibers have an approximate axial symmetry, it is reasonable to assume that the two smaller eigenvalues of diffusion tensors are equal. In practice, we fix the eigenvalues of σ_i to specified values (1.5, 0.4, 0.4) μ^2/ms consistent with the values commonly observed in white-matter tracts. The principal directions of σ_i are then evenly distributed on the unit sphere. Note that the number of components in mixture only depends on the resolution of sphere tessellation and should not be interpreted as the expected number of fiber bundles. Hence only the weights w_i remain to be estimated and can be solved using a spherical deconvolution approach [4] with a regularization scheme.

After the mixing distribution is obtained in the form $dF = \sum w_i d\gamma_{pi,\sigma i}$, the displacement PDF can be approximated by taking the Fourier transform

$$P(r) = \int_{R^3}\int_{P_n} \exp(-q^T Dqt)\,dF(D)\exp(-iq\cdot r)\,dq \approx \sum_{i=1}^{N} \frac{w_i}{\sqrt{(4\pi t)^3\,|\hat{D}_i|}} \exp\left(-\frac{r^T\hat{D}_i^{-1}r}{4t}\right)$$

where $\hat{D} = p\sigma_i$ are the expected values of $\gamma_{p,\sigma i}$.

RESULTS AND DISCUSSION

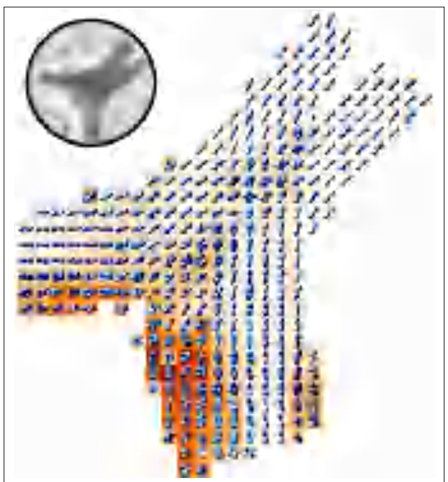

Fig. 1 A HARDI dataset from an excised rat optic chiasm was acquired at 14.1T using Bruker Avance imaging systems. The data set consisted of 46 directional DWIs with a b-value of 1250s/mm^2 along with a single image acquired at b = 0s/mm^2. The figure shows the displacement probabilities computed from this image. For the sake of clarity, we excluded every other pixel and overlaid the probability surfaces on generalized anisotropy (GA) maps.

We presented a new mathematical model for the diffusion-weighted MR signals obtained from a single voxel yielding an explicit form of the expected MR signal attenuation characterized by a Rigaut-type asymptotic fractal formula. Using this new model in conjunction with a deconvolution approach, we have presented an efficient estimation scheme for the distinct fiber orientations and the water molecular displacement probability functions at each lattice point in a HARDI data set.

Figure 1 shows the displacement probabilities computed from an optic chiasm region of excised, perfusion-fixed rat nervous tissue. As evident from this figure, our method is able to demonstrate the distinct fiber orientations in the central region of the optic chiasm where ipsilateral myelinated axons from the two optic nerves cross and form the contralateral optic tracts.

Because the end result is expressed as a mixture of oriented Gaussians, many of the quantities produced by other methods (such as the orientation distribution function, probability profile, fiber orientation distribution,… etc.) are easily computable using our technique. This observation provides an important opportunity to understand these quantities and evaluate their performances in resolving complicated structures. Figure 2 demonstrates such a comparison where all of the profiles were computed using the method presented in this work.

Fig. 2 The figure on the right shows the results on simulated signal attenuation from cylindrical tubes. For each of the 1-, 2- and 3-fiber profiles, the following quantities computed using the described method are presented from top to bottom: (1) the displacement probability P(|r|=15μm) as in DOT [5]; (2) the radial integral of P(r) as in QBI [6]; (3) the integral of P(r)r^2 as in DSI [7]. It is evident that all three selected quantities based on our model are able to capture the fiber orientations, yet the constructed probability surfaces P(r) exhibit the sharpest local structure.

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